

A graphical foundation for schedules

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Game semantics

- ✦ Highly represented at MFPS 2012
- ✦ Needs no further introduction from me

You may have noticed...

- Heavy use of pictures to describe structure
- Interleaving in plays
- Compositional structure
- Pointers and justifications

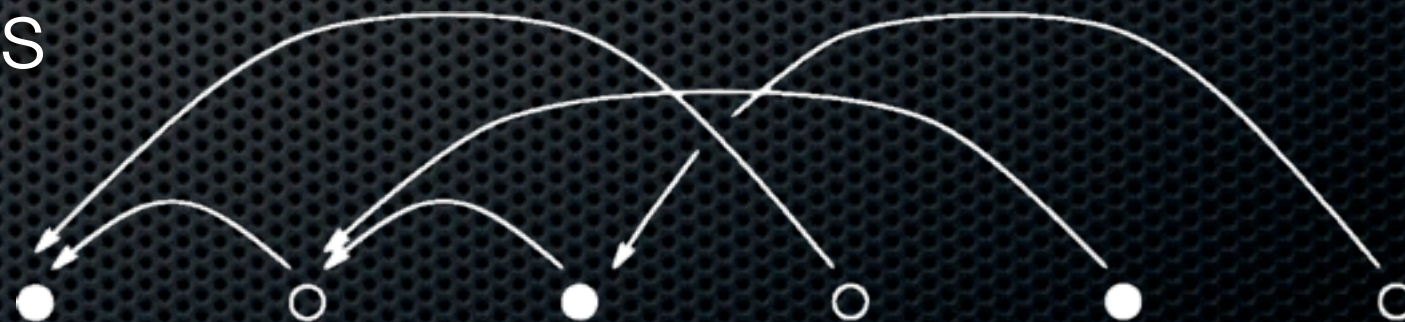
$$\mathbb{B} \rightarrow \mathbb{B}$$

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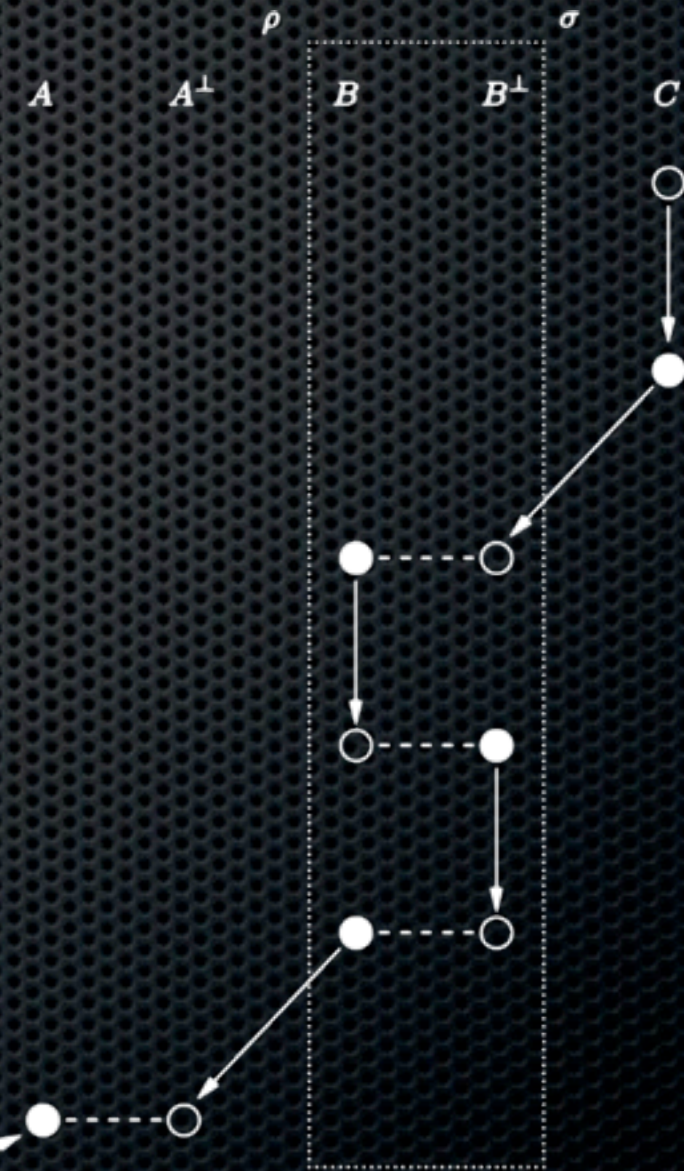
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(Hyland and Ong 2000)



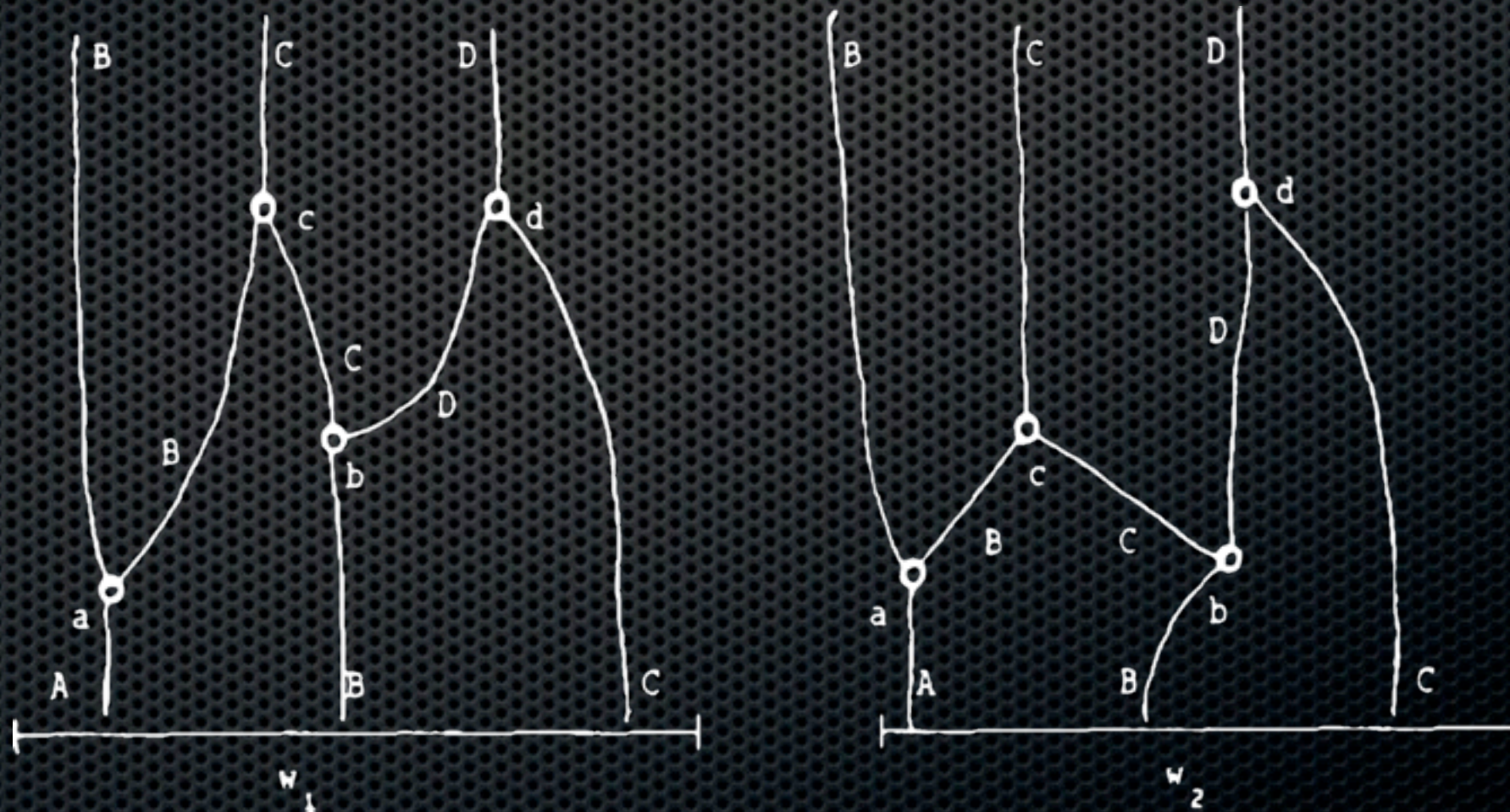
(Schalk 2001)

Why are pictures used?

- ✦ Definitions combinatorial and complicated
- ✦ Diagrams drawn capture structure successfully
 - ✦ Composition “works”
 - ✦ There’s some categorical similarity to be investigated
- ✦ “Pictures are used because *they can be*”

Not a unique phenomenon

- String diagrams for tensor algebra
 - Joyal and Street formalised this topologically (1991)



Schedules

- A *schedule* is a combinatorial device describing interleaving
- Originally from Harmer et al.'s 2007 paper
 - Binary strings
 - Order relations

$\mathbb{B} \rightarrow \mathbb{B}$

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Schedules

- ✦ There's a notion of composition
 - ✦ It's complicated
 - ✦ It's associative
 - ✦ It has a unit (copycat)
- ✦ Schedules form a category

$\mathbb{B} \rightarrow \mathbb{B}$

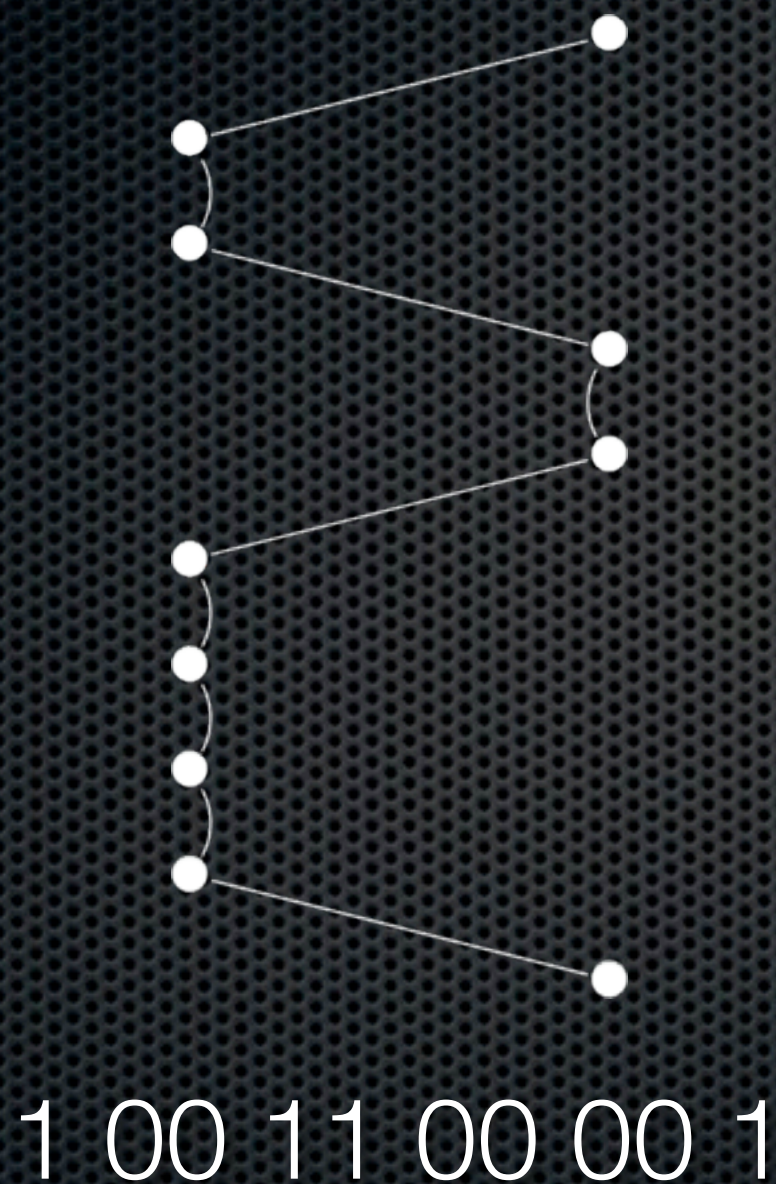
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Schedules: in practice



- ✦ Draw plays with moves in respective components
- ✦ Positions, lines encode combinatorial structure
- ✦ Why do it this way?
 - ✦ Easy to consider
 - ✦ Composition is easy

Schedules: composition

- ✦ Definition:
combinatorial
bookkeeping
- ✦ Pictures convince
audiences, readers

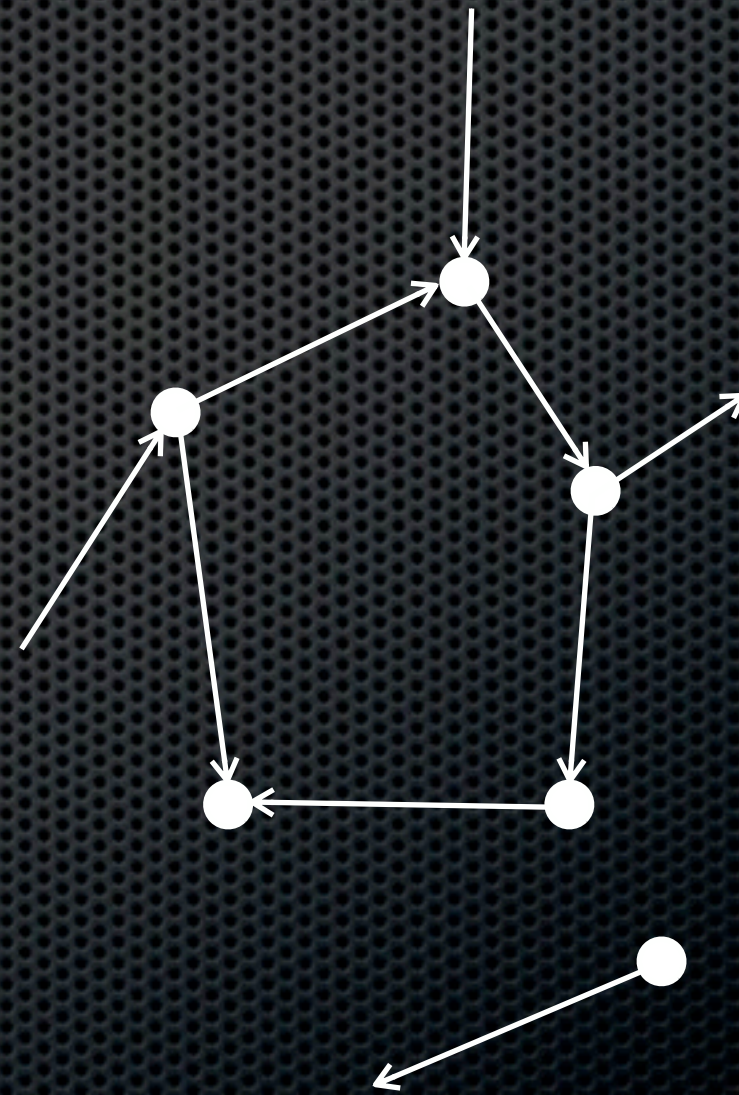


Schedules: the key point

- ✦ This use of pictures *works*
- ✦ There's a correspondence between
 - (1) What people think and draw
 - (2) The mathematical formalism
- ✦ This correspondence needs to be made precise

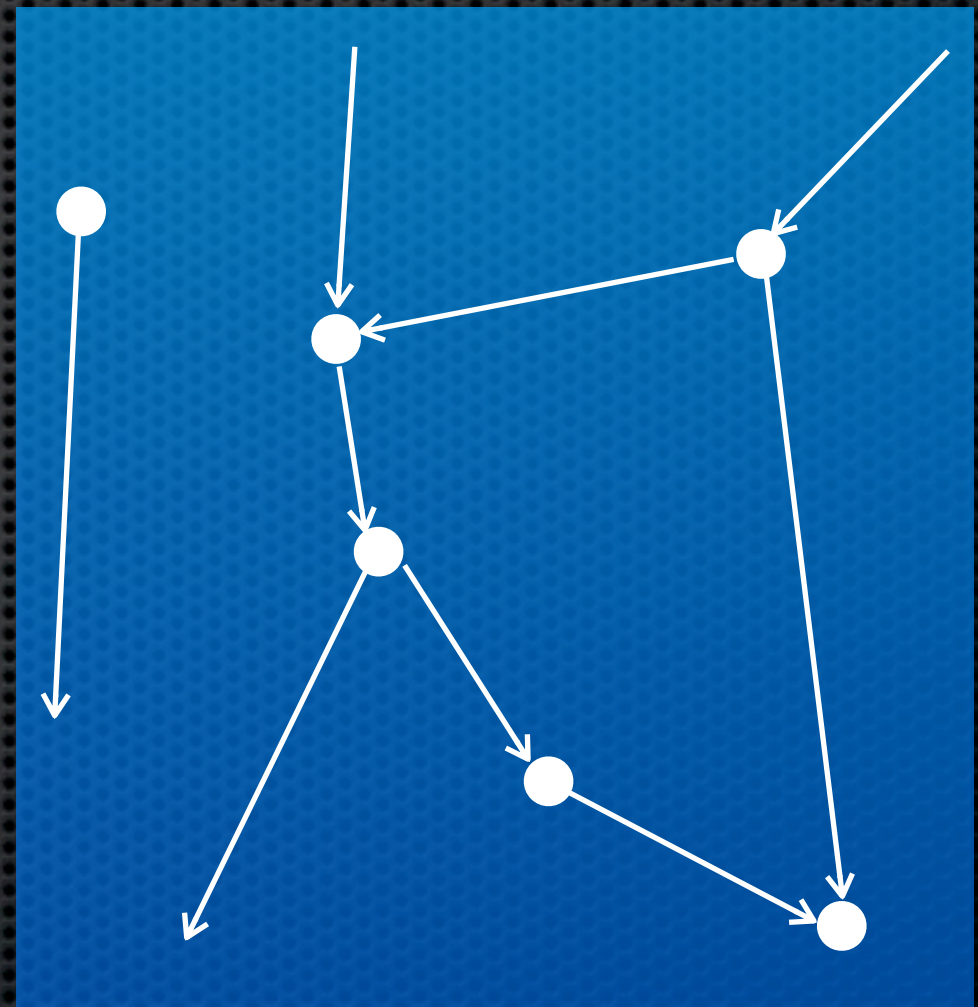
Graphical foundation

- ✦ **Progressive graph** (À la Joyal and Street):
 - ✦ Hausdorff space
 - ✦ Distinguished **nodes**
 - ✦ Nodes separate space into **edges**
 - ✦ No cycles

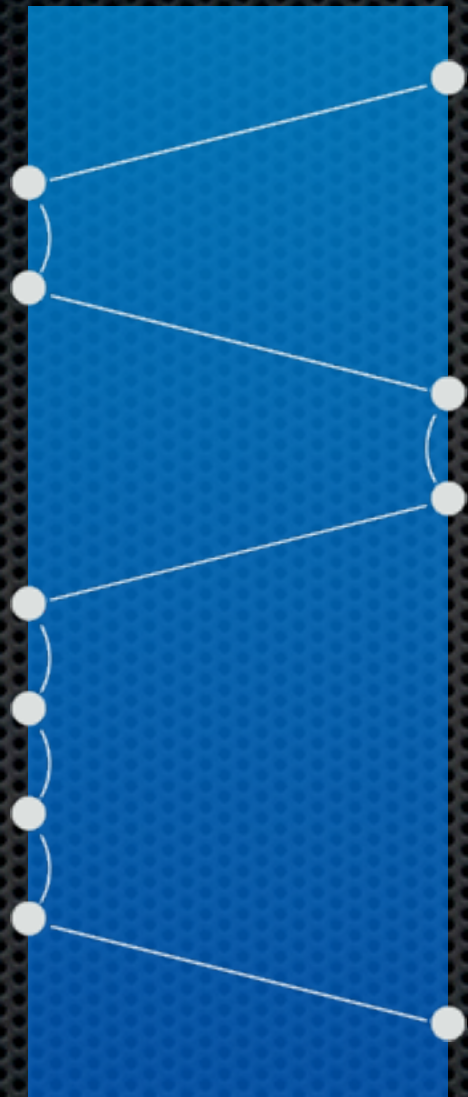


Graphical foundation

- ✦ **Embedding:**
 - ✦ Injective map into the plane
- ✦ **Progressive:**
 - ✦ Second projection injective on edges
 - ✦ Edges point down



Graphical foundation



- ✦ **Schedule:**

- ✦ Path in plane
- ✦ Ordered nodes
- ✦ Left-right positions important
- ✦ (Extra conditions)

Graphical composition

- ✦ Simple procedure to find correct sub-graph
 - ✦ Left-right alternating path
- ✦ Alternatively: characterised by a Hamiltonian path

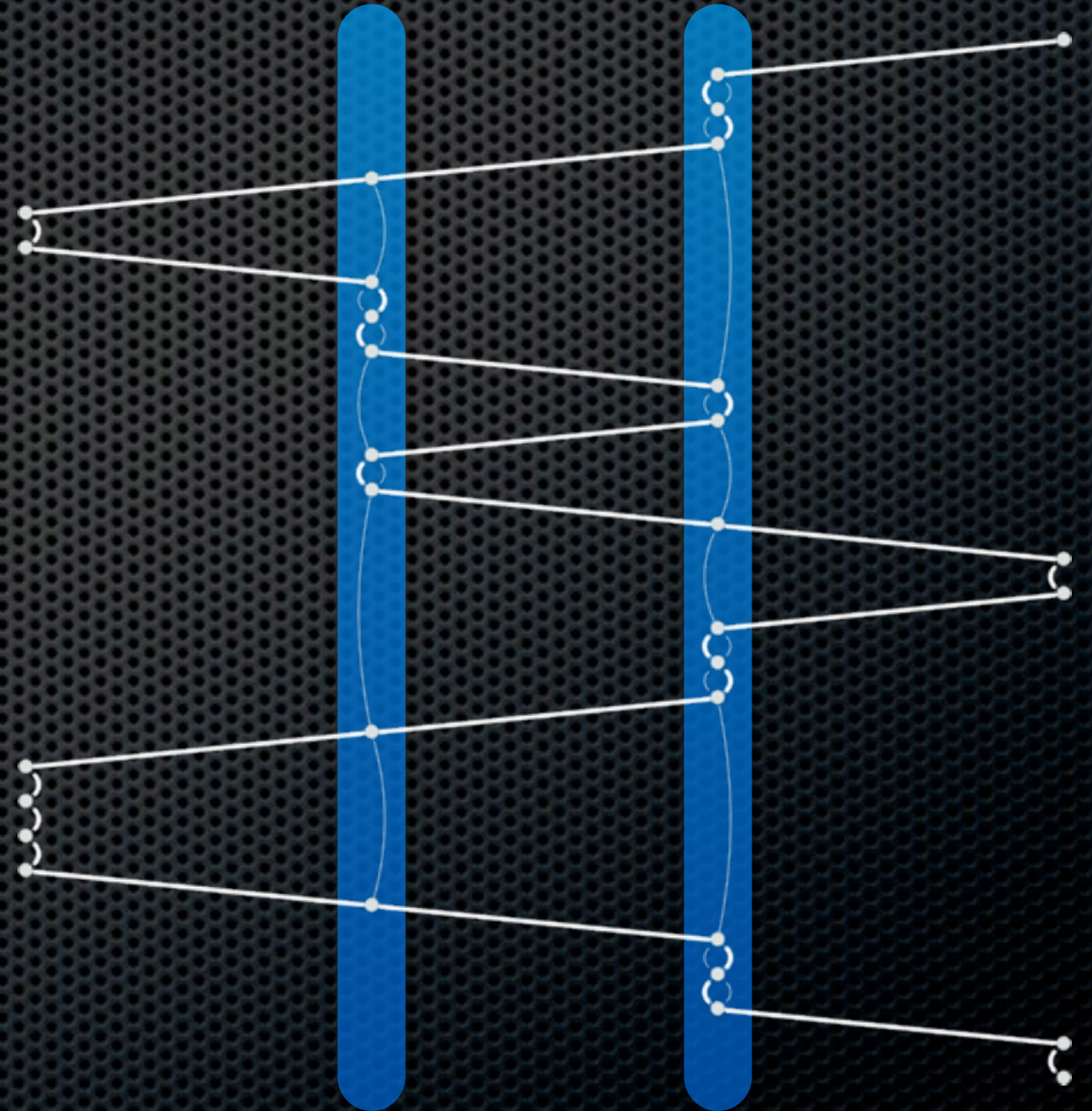


The category *Sched*

- ✦ Objects
 - ✦ Natural numbers
- ✦ Morphisms
 - ✦ Topological entities — plane graphs up to deformation
 - ✦ What people draw
- ✦ Composition
 - ✦ A graphical procedure
- ✦ Key properties required
 - ✦ Associativity
 - ✦ Identities (copycat)

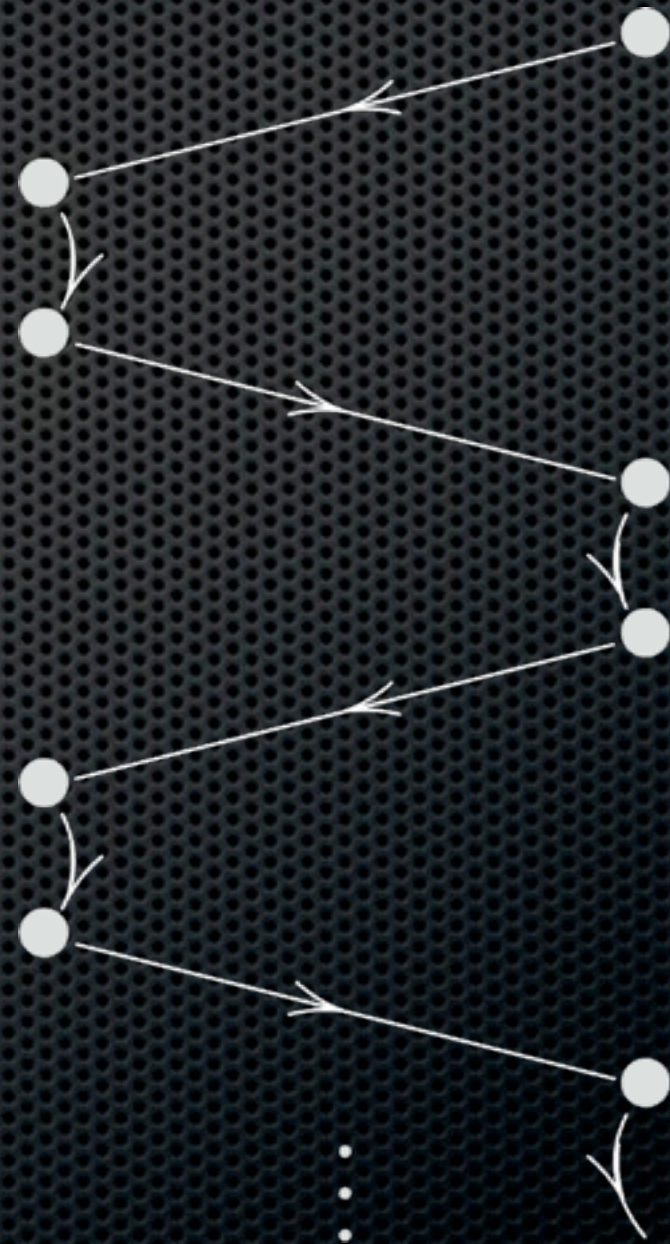
Graphical associativity

- ✦ Proof is one-line
 - ✦ Equivalent to remove left-hand or right-hand edges first
- ✦ “It’s associative because planar juxtaposition is”



Graphical identities

- ✦ Copycat
- ✦ Proof just as straightforward

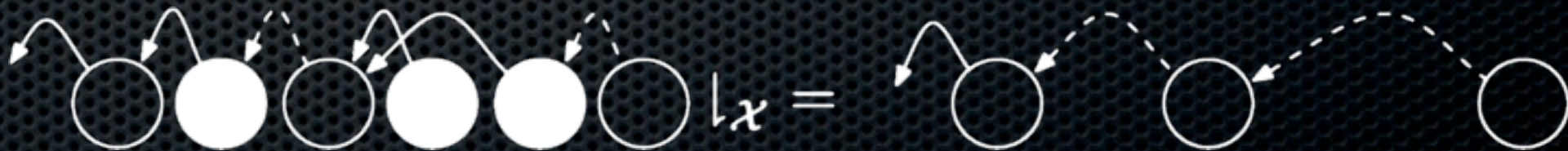


Furthermore...

- ✦ *Sched* is equivalent to Harmer et al.'s category of schedules
- ✦ We can do schedule composition via pictures
 - ✦ No loss of rigour

Further research

- ✦ Look at where else in game semantics pictorial methods are employed
 - ✦ For examples...
 - ✦ Pointers/heaps
 - ✦ Trees for exponentials
 - ✦ Develop further methods
 - ✦ Explore categorical structures
 - ✦ Give new categories of games and strategies?



(Gabbay and Ghica 2012)